FACTOR DEMAND FUNCTIONS FOR CANADIAN INDUSTRIES, 1946 - 1969

by
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and
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DEPARTMENT OF MANPOWER AND IMMIGRATION
CANADA

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The study "Factor Demand Functions for Canadian Industries, 1946-69" is a first attempt at estimating cost (or equivalently production) functions for the Canadian economy. These functions based on several classes of capital and one class of labour have been estimated for ten industrial sectors, using the Generalized Leontief functional form due to Diewert [1]. The study likely gives, for the Canadian economy, the first quantification of production functions which allow estimation of the Hicks-Allen partial elasticities of substitution between all pairs of input factors, at this level of disaggregation.

The study was prepared by A.D. Woodland, Assistant Professor of Economics at the University of British Columbia under contract to the Department of Manpower and Immigration.

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FACTOR DEMAND FUNCTIONS FOR

CANADIAN INDUSTRIES, 1946-1969

by

A.D. Woodland*

1. Introduction

Recent years have seen rapidly increasing interest in the development of new functional forms which may be used to describe production functions. These functional forms provide an escape from the narrow class of CES (constant elasticity of substitution) functions which are particularly restrictive when the number of factors exceeds two. While the new functional forms provide increased substitution possibilities, the number of parameters is much greater and this reduces degrees of freedom in their estimation.

Associated with the development of new functional forms for production function has been renewed interest in the duality between production and cost functions. This duality, developed by Shephard (1953, 1970) and Uzawa (1964), is useful for present purposes in that it says that if we have a functional form for a cost function satisfying certain conditions (such as concavity, linear homogeneity in factor rewards, monotonicity), then there exists a production function which satisfies all of the conditions imposed by economic theory and which has as its corresponding cost function the cost function we began with. Thus we may specify a functional form for a cost function knowing that there is a well behaved production function, perhaps not analytically tractable, underlying it. We only have to assume that the producer is a factor reward taker and a cost minimiser.

If the cost function is differentiable then the factor demand functions may be easily obtained from the cost function as the partial derivative functions with respect to factor rewards. This result is known as Shephard's or Hotelling's Lemma,[1] which, in conjunction with the duality between production and cost functions permits us to by-pass the production function and use the cost function alone in deriving factor demand functions.

In this paper an attempt is made to estimate demand functions for factors of production using the theory outlined above. This attempt is important in several respects. First, more than two factors of production are considered. At this stage of the project we consider just one labour type and, depending upon the industry, from two to four capital types. Thus it is an attempt to move away from the one capital one labour division of factors. Second, application to 10 industrial divisions is attempted. Usually applications have been restricted to the manufacturing sector which is an important but clearly not the only source of demand for factors. Third, it provides a test of the proposition that there exists an aggregate (industry division level) production function having the properties of an individual production function. Fourth, it provides tests of various restrictions on the nature of the production function. Finally, estimates of the direct and cross elasticities of demand for factors with respect to factor rewards are obtained.

2. THEORY

In this section we briefly describe the method used in the estimation of the factor demand functions. First we specify a functional form for the cost function, then derive the factor demand functions and various other functions, and finally we indicate the method of statistical estimation.

The cost function is assumed to have the following form essentially due to Diewert (1971):

2.1 $$C(w,y,z) \equiv y\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}(w_i w_j)^{\frac{1}{2}} + y\sum_{i=1}^{n}\beta_{i,n+1}w_i + z\sum_{i=1}^{n}\beta_{i,n+2}w_i\right]$$
$$\beta_{ij}=\beta_{ji} \quad i,j=1,\ldots\ldots,n.$$

where y is the output level, z is an index of technology, $w = (w_1,\ldots,w_n)$ is a vector of factor rewards and the β_{ij} $(i=1,\ldots,n;\ j=1,\ldots,n+2)$ are parameters. The factor demand functions, obtained by partially differentiating with respect to factor prices, are

2.2 $$x_i(w,y,z) = y\left[\sum_{j=1}^{n}\beta_{ij}(w_j/w_i)^{\frac{1}{2}} + \beta_{i,n+1}y + \beta_{i,n+2}z\right] \qquad i=1,\ldots,n.$$

In terms of input-output coefficients these functions are

2.3 $$a_i(w,y,z) = \sum_{j=1}^{n}\beta_{ij}(w_j/w_i)^{\frac{1}{2}} + \beta_{i,n+1}y + \beta_{i,n+2}z \qquad i=1,\ldots,n.$$

where $a_i \equiv x_i/y$ is the input of factor i per unit of output.

It should be noted that the cost function is homogeneous of degree one in factor rewards, and if $\beta_{i,n+1} = 0$ $i=1,\ldots,n$ it is homogeneous of degree one in output as well. Also if $\beta_{ij} \geq 0$ for all $i,j=1,\ldots,n,\ i\neq j$ then concavity in factor rewards and monotonicity hold over the whole domain of positive factor rewards.

The elasticities of demand for factors with respect to factor rewards are

2.4 $$\varepsilon_{ij}(w,y,z) = \frac{\frac{1}{2}\beta_{ij}(w_j/w_i)^{\frac{1}{2}}}{a_i(w,y,z)} \qquad i\neq j \quad i,j=1,\ldots,n.$$

$$\varepsilon_{ii}(w,y,z) = -\sum_{\substack{j\neq i\\ j=1}}^{n}\varepsilon_{ij}(w,y,z) \qquad i=1,\ldots,n.$$

while the Hicks-Allen elasticities of substitution are

2.5 $$\sigma_{ij}(w,y,z) = \varepsilon_{ij}(w,y,z)\,/\,\theta_j(w,y,z) \qquad i,j=1,\ldots,n.$$

where $\theta_j(w,y,z) \equiv w_i a_i(w,y,z) \,/\left\{\sum_{j=1}^{n} w_i a_i(w,y,z)\right\}$ is the share of factor i in the total cost of producing output y.

The cost function being a second order approximation to any cost function, can at the point of approximation describe any arbitrary matrix of substitution elasticities. Furthermore, given the parameters, the elasticities vary around the isoquants and from one isoquant to another.

For estimation purposes the relationship between observed factor rewards, output, and the technology index and the input-output coefficients is assumed to be

$$a_{it} = \sum_{j=1}^{n} \beta_{ij}(w_{jt}/w_{it})^{\frac{1}{2}} + \beta_{i,n+1}\, y_t + \beta_{i,n+2}\, z_t + u_{it} \qquad \begin{array}{l} i=1,\ldots,n. \\ t=1,\ldots,T. \end{array}$$

where the subscript t denotes the t^{th} observation and u_{it} is a random error term. The stochastic assumptions are

$$u_t = (u_{1t},\ldots,u_{nt}) \sim N(0\ ,\ \Sigma) \qquad t=1,\ldots,T.$$

$$E(u_t^{`} u_s) = \begin{cases} 0 & t \neq s \quad s,t=1,\ldots,T. \\ \Sigma & t=s \end{cases}$$

where E is the expectation operator and Σ is a positive definite matrix of contemporaneous covariances between the n disturbance terms. That is, we assume away the existence of autocorrelated disturbances but allow non-zero contemporaneous covariances between the disturbance terms in the different equations. We also assume that the explanatory variables w,y,z are independent of the disturbance terms which by-passes the simultaneous equation problems. Justification for this is more clearly established at the industrial level than at the national level if we believe that the factor markets are nation wide and that no one industry can exercise monopoly power in the factor market. Then we can treat each industry as a price taker.

Since the demand functions are connected via the covariance specification and by the symmetry constraints the equations should be estimated jointly using Zellner's (1962) generalised least squares method.[2]

Our procedure is as follows: Estimate 2.6 using $\Sigma = I$ without imposing the symmetry restrictions; use the residuals to compute a consistent estimator $\hat{\Sigma}$ for Σ; apply generalized least squares to 2.6 once again but now using $\hat{\Sigma}$ as the covariance matrix. This provides us with estimates for the parameters of the demand functions.

Clearly these estimates may not satisfy the symmetry condition imposed on the parameters by the cost function. In fact, given the parameters β_{ij} of the demand functions we may ask whether they could have been derived from a cost function. The answer is yes if and only if the symmetry condition is satisfied. Thus a test of the symmetry condition is a test for the existence of a cost function and hence, indirectly, for the existence of a production function. This test, carried out using the normality assumption via the F distribution, is strictly applicable only if Σ is known but may be applied as an approximate test using asymptotic theory. This applies to all our tests.

It is of some interest to know whether the input-output coefficients are independent of the level of output. In the symmetric case the independence implies constant returns to scale in production. At the aggregate level we know that identical individual production functions, the possibility of "in-action", and the existence of free entry into a perfectly competitive industry implies that the "aggregate production function" will exhibit constant returns to scale. By testing for homogeneity we are effectively testing this collection of assumptions. The test is whether or not $\beta_{i,n+1} = 0$ for all i. Clearly, if symmetry is not assumed, the test cannot have this same interpretation since then a production function does not exist.

The nature of technological change may also be examined within this model. Our index is very crude being simply a linear time variable. The hypothesis that technological change has been zero may be tested by testing whether $\beta_{i,n+2}=0$ for all i. This test may be applied whether or not symmetry is imposed.

Finally, we may wish to test whether the input-output coefficients are independent of factor rewards. In this case symmetry is trivially forced so it is a test of whether the production function exhibits substitution possibilities as opposed to a Leontief technology. Here we test the restriction that $\beta_{ij} = 0 \quad i \neq j \quad i,j=1,\ldots,n$.

The various restriction tests are illustrated schematically below

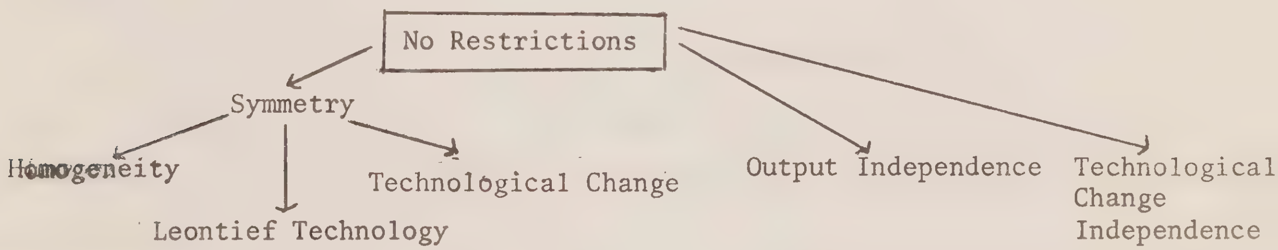

FIGURE 1

3. Data

The Canadian economy was divided into divisions according to the 1960 Standard Industrial Classification. There are 11 such divisions: Agriculture; Forestry; Fishing and Trapping; Mining; Manufacturing; Construction; Transportation, Communication, Utilities; Trade; Finance; Services; Government. In this study we consider the first 10 of these divisions over the period 1946-1969 using annual data.

In order to estimate the factor demand functions we need observations on output quantity, factor input quantities, factor input rewards, and an index of technological change for each of the 10 industries. In the absence of a suitable alternative the index of technological change was taken to be time.

3.1 Output

The revised National Accounts[3] 1926-1969 provide annual time series for Gross Domestic Product at Factor Cost, by Industry in Table 28. Also Statistics Canada provides Indices of Real Domestic Product, by Industry in publications 61-506 for the period 1936-1960: 61-510 for the period 1961-1967, and 61-005 (Annual supplement to the May 1971 issue) for the period 1968-70. Applying the output indices to the value series we get a series for Real Gross Domestic Product at Factor Cost, by Industry in terms of 1961 (the base year for the indices) prices. This we use as an index of output in the industry.

3.2 Labour

There is assumed to be just one homogeneous labour type in this study. The numbers of workers in each industry is provided by the Labour Force Survey. Statistics Canada also has unpublished data on average hours worked per week by workers in each industry. Assuming that the number of weeks worked per year is constant at 50 weeks we can obtain time series for the number of hours worked per year in each industry. This is our labour input quantity variable.

The wage bill in each industry is obtained as follows. From the 1961 census ratios of self-employed workers to total number of workers of occupation i in industry j were calculated. Assuming that these ratios applied throughout the 1946-1969 period, and using unpublished estimates[4] of the number of workers of type i in industry j in each year, a final ratio of self-employed to total workers in each industry was obtained. Then assuming that the wage bill provided on an industry basis in the revised National Accounts 1926-1969 in Table 29 as "Wages, Salaries,

and Supplementary Labour Income, and Military Pay and Allowances" applies to those workers not self-employed, and that all workers receive equal renumeration, the wage bill in the Revised National Accounts was "blown up" to get a total wage bill.

On the grounds that such a calculation has greater chance of being correct at the aggregate level due to some inadequacies in the industrial and occupational classifications (eg. mixing of the "establishment" and the "enterprise" concepts) the ratio of aggregate self-employed earnings to aggregate unincorporated earnings was assumed to be the same for all industries. This ratio was applied to unincorporated earnings in each of the industries except Finance to get self-employed earnings which were added to the national accounts wage bill to get a total wage bill. Since the imputed rental on owner occupied dwellings is included in unincorporated earnings in Finance, the original estimated wage bill was used for this industry.

The wage rate per hour was obtained by dividing the estimated wage bill by the number of hours worked in each industry.

3.3 Capital

Due to data problems only fixed capital was considered, inventories and land being ignored. Admittedly, this generates a specification bias in the results and also creates problems in the calculation of rentals on fixed capital.

The basic source of information was the Statistics Canada project on Capital Stocks. Some of this information has been published in publications 13-522 and 13-543 for the manufacturing sector. The data used here is unpublished[5]. The information consists of a purchase price index, a gross investment series, and a real net capital stock series for the period 1926-1970

for various types of fixed capital in all industries. This information often had to be aggregated to the 10 industrial divisions used in this study.

Generally four capital types are defined: building structures (BS), engineering structures (ES), machinery and equipment (ME), and operating capital (OC). In some industries special capital types were defined. In Agriculture passenger vehicles (PV) and commercial vehicles (CV) as well as farm machinery (FM) were defined while in Fishing vessels (V), boats (B), and gear (G) were defined.

To obtain depreciation rates we note the procedure whereby Statistics Canada estimated their average economic lives (see 13-522 page 88) and simply reverse this procedure. Thus we compute the geometric depreciation rate as $\delta = 2/L$ where L is the average economic life provided by Statistics Canada. For some industrial divisions an average depreciation rate was calculated.

The capital stocks used in the present study were calculated assuming a geometric depreciation rate according to the formula

3.1 $$K_t = I_{t-1} + (1-\delta)\ K_{t-1} \qquad t=1,...$$

where I_t is real gross investment, δ is the depreciation rate common to all vintages of capital, and where K_o the base stock was taken to be the Statistics Canada stock at the end of 1926. Note K_t is the stock at the beginning of period t and we assume additions to that stock over the year are not used in the production process.

The imputed rentals on capital are obtained as follows. A capitalist can purchase one unit of capital at the beginning of period t for Q_t dollars and then rent it to a firm for one year collecting a rental of R_t dollars. At the beginning of the next period only $(1-\delta)$ units will be left because depreciation will have taken δ, so he expects the capital to be worth $(1-\delta)\ \hat{Q}_{t+1}$ where $\hat{Q}_{t+1}$ is the price he expects at the beginning of period t+1. In addition he has to pay taxes on his taxable earnings at the rate u_t, his expected taxable earnings being $R_t - v_t \delta\ \hat{Q}_{t+1}$ where v_t is the proportion

of depreciation allowable for taxation. Thus the expected net value of buying the capital good and then selling it next period is

$$R_t + (1-\delta)\ \hat{Q}_{t+1} - u_t \{R_t - v_t\ \delta\ \hat{Q}_{t+1}\} - Q_t$$

so the expected rate of return is

3.2 $$r_t = \frac{R_t + (1-\delta)\ \hat{Q}_{t+1} - u_t \{R_t - v_t\ \delta\ \hat{Q}_{t+1}\} - Q_t}{Q_t}$$

Given a rate of return r_t, the purchase prices Q_t and $\hat{Q}_{t+1}$ and the parameters δ, u_t, v_t we can calculate the rental R_t which will yield this rate of return r_t. We assume owners of capital have some rate r_t in mind when rentals are formed - in fact if everyone uses the same r_t then competition in the capital market will alter rentals until the equality in 3.2 is attained.

Thus we rewrite 3.2 as

3.3 $$R_t = \frac{r_t\ Q_t - (\hat{Q}_{t+1} - Q_t) + \delta\ \hat{Q}_{t+1} - u_t\ v_t\ \delta\ \hat{Q}_{t+1}}{1-u_t}$$

It is assumed that r_t is the ex post rate of return on all capital types within the industry in question. If the capital types are subscripted by i then we form r_t as follows

3.4 $$r_t = \frac{\sum_i R_{it}\ K_{it} + \sum_i (\hat{Q}_{i,t+1} - Q_{i,t})\ K_{it} - \sum_i \delta_i\ Q_{i,t+1}\ K_{it} - u_t \left[\sum_i R_{it}\ K_{it} - v_t \sum_i \delta_i\ \hat{Q}_{i,t+1}\ K_{it}\right]}{\sum_i Q_{it}\ K_{it}}$$

To calculate the rate of return r_t each of the terms in 3.4 need to be evaluated. For present purposes it is assumed that price expectations are static so that we may set $\hat{Q}_{i,t+1} = Q_{i,t}$. The term $\sum_i R_{it} K_{it}$ is taken to be the rental bill in the industry in question, obtained by substracting the estimated wage bill from gross domestic product at factor cost. This attributes all rental earnings to fixed capital an overstatement since land and inventories are ignored. The tax rate u_t is computed as a weighted average of the corporation tax rate (in aggregate) to the personal tax rate the weights being the proportion of rental earnings attributed to corporations and minus this proportion. The revised National Accounts provided the information required for this calculation. The term $v_t \sum_i \delta_i \hat{Q}_{i,t+1} K_{it}$ was taken to be the capital consumption allowance in the industry in question, this unpublished information being made available by Statistics Canada. The remaining terms were computed using above mentioned data. This imputed rate of return was then used in 3.3 to compute imputed rentals.

This completes the data description. In summary, for each industry (division) we have a series on output, wage rates, hours worked, rentals on each capital type, and capital stocks for each capital type. Prior to estimation the data was adjusted such that all factor rewards were normalised to unity in 1961.

4. Results

For each industry we estimated the factor demand function system by ordinary least squares and used these results to compute an estimated covariance matrix for the residuals. This estimated covariance matrix was then used to estimate the demand system by generalised least squares under alternative parameter restriction. First, no restrictions were imposed; second, symmetry restrictions were imposed; third, symmetry plus no output coefficients; fourth symmetry, no output coefficients, no time coefficients; fifth, no output coefficients; sixth,

no time coefficients; seventh, no factor rewards coefficients. Table 1 provides a summary of the results in terms of the various restrictions by showing the calculated F values for the test of the hypothesis that a restriction set is valid. Below the F value are the degrees of freedom in the numerator (equals the number of restrictions) while the degrees of freedom in the denominator are provided in the last column. Beside the F value, in parenthesis, is the level of significance at which the null hypothesis is accepted. No such value means that the F value was significant even at the .001 level.

4.1 Symmetry

As indicated in Table 1 the symmetry restrictions on the parameters were rejected in all industries except three - Forestry (2), Transportaion (7) and Trade (8). This implies that, conditional upon our data and model specifications, we cannot accept the hypothesis that there exists an aggregate industry production function for the remaining seven industries. Of course, various explanations may be offered. First, we are aggregating over disimilar sub-industries producing different products with different production functions. In such cases the aggregate demand functions for labour will be mis-specified. This is true even if each sub-industry has the same functional form for the production hence cost function. Second, our use of value added as the output variable loses its validity if the prices of the outputs and intermediate inputs do not move in direct proportion to each other, for then we are not able to make use of Hicks' composite good theorem. Third, it may be that costs are not being minimised so that individual firms are not on their theoretical cost functions. One reason is that full adjustment of actual stocks of capital to desired stocks takes time so that firms are often heading towards their desired stock but are seldom there for very long. This possibility has not been examined in this study. Fourth, we may have mis-specified the functional form or may have ommitted factors which should have been included or

Table 1 : F-values for Restriction Tests

Industry	Symmetry	Symmetry Output	Symmetry Output, Time	Output	Time	Prices	
1	16.93 10	63.78 15	62.35 20	147.19 5	63.64 5	17.39 20	85
2	2.09 (.05) 6	17.60 10	75.28 14	35.39 4	55.68 4	2.62 .01 12	72
3	23.60 10	18.08 15	15.23 20	15.34 5	17.21 5	37.37 20	85
4	14.22 6	9.55 10	17.85 14	0.988(.1) 4	0.45(.1) 4	25.31 12	72
5	14.05 10	32.01 15	108.86 20	19.60 5	16.96 5	15.46 20	85
6	6.97 6	7.86 10	37.60 14	3.95(.001) 4	6.69 4	9.30 12	72
7	2.83(.001) 10	17.97 15	116.89 20	15.41 5	6.70 5	9.25 20	85
8	2.36(.01) 10	29.24 15	43.41 20	36.91 5	19.48 5	2.92 20	85
9	52.76 3	67.72 6	337.51 9	8.42 3	33.78 3	29.41 6	57
10	17.47 3	32.83 6	180.60 9	50.95 3	451.93 3	13.37 6	57

(α) means not significant at the α level but significant at the next highest level. The levels considered are 0.001 , 0.01 , 0.05 , 0.10 .

may have estimated the factor rewards and input quantities incorrectly. Whatever the reason, given the data base, we should be hesitant to treat industries as if they have a well behaved production or cost function analogous to that of the individual plant.

Suppose that we ignore the results just obtained and simply assume that aggregate production functions exist and carry out our estimation under this maintained hypothesis. Then it is appropriate to impose symmetry restrictions right from the beginning before the estimation of the covariance matrix for the disturbance terms. This was done for the ten industries and the results are rather disturbing.

In all industries except Forestry (2), Finance (9), and Services(10), the estimated demand functions yielded negative estimated input-output coefficients for most observations of factor rewards **output and time** for at least one and often two factors of production. In these industries there were "bad" estimates for some of the remaining factors meaning that the inputs were consistently over or under-estimated. All this, despite a very good fit in terms of the "transformed variables". The reason seems to be as follows. Under generalised least squares different weights are given to the errors in the various factor demand equations. At the same time the equality of parameters across equations (symmetry) is forced. Thus some coefficients are effectively determined by one factor equation given high priority, then imposed on another equation whose errors are given low priority. Thus there is the possibility, more than adequately demonstrated in the present empirical results, that the fit for one or more equations is very bad while the fit in some equations, given high priority, is very good. Here the good fit was given to labour in general.

Imposing the symmetry conditions before estimating the covariance matrix must have itself generated a covariance matrix which gave wide differences in weights to the various factors. In comparison, when symmetry was

imposed after the covariance matrix had been estimated from ordinary least squares, the input-output coefficient estimates were never negative and always "good" providing reasonably close approximations to the observed input-output coefficients.

If this explanation was correct, we would expect, on the basis of Table 1, that industries 2,7, and 8 would yield good fits under the maintained hypothesis of symmetry. However, only industry 2 (Forestry) in this group yielded good results while two industries outside this group - 9 (Finance) and 10 (Services) - yielded good results.

One other feature of the results should be noted. The unconstrained generalised least squares estimates showed that quite often sign $\hat{\beta}_{ij} \neq$ sign $\hat{\beta}_{ji}$ both $\hat{\beta}_{ij}$ and $\hat{\beta}_{ji}$ being significant.

4.2 Other Restriction Sets

Clearly when symmetry is rejected the addition of the homotheticity restriction and then the addition of no technological change to the symmetry restrictions will generally also lead to rejection of these enlarged restriction sets. The results are shown, for completeness, in columns 2 and 3 in Table 1. Even where symmetry was accepted, symmetry and homotheticity were rejected convincingly. If we test for homogeneity under the maintained hypothesis of symmetry given the estimated covariance matrix estimated without symmetry, then we can accept homogeneity of the production function for Fishing at the .05 level and for Mining at the .1 level. In all other cases homogeniety is rejected at the .001 level of significance. Thus in most cases we must reject the collection of hypotheses - identical technologies for firms, the possibility of inaction, perfect competition, and free entry - used to justify constant returns to scale at the industry level of aggregation. This rejection is, of course, conditional upon our data and model.

Column 4 of Table 1 shows the calculated F values for testing the hypothesis that the input-output coefficients are independent of the output level without symmetry being imposed. This hypothesis is acceptable in only two industries, these being Mining (4) and Transportation (6) and in the latter only at the .001 level of significance. Thus on the whole it seems that the scale of operations does play an important role in determining the input-output coefficients.

The following remarks refer to the symmetry constrained estimates. In Agriculture (1), Trade (8) and Forestry (2) there appear to be economies of scale with respect to the inputs. An examination of the output coefficients (See Appendix) shows that the significant ones are all negative for these industries indicating that the input-output coefficients decline as output increases. In Mining (4) and Finance (9) there are diseconomies of scale with respect to all inputs with significant output coefficients. In the remaining industries the effect of the output level has differential effects on the various factors. In Manufacturing (5) for example, optimal techniques become more labour intensive as output expands, while there appear to be economies in the use of building and engineering structures and machinery and equipment.

Column 5 of Table 1 indicates that we must accept that technological change has occurred in all industries except Mining. The only surprise here is that the F value for Mining was not significant even at the 0.1 level. Under symmetry the labour-output coefficient in Forestry (2), Fishing (3), Mining (4), Construction (6), Transportation (7) and Finance (9) has declined significantly over time due to factors other than prices and output and which we might call technological change. Only in Agriculture (1), Manufacturing (5) and Services (10) has the labour coefficient increased significantly.

Finally we test the hypothesis that the input-output coefficients are independent of the factor prices (service prices) which is the hypothesis that the production function is "Leontief". This hypothesis is rejected at the .001 level for all industries except Forestry (2) and for this industry it is rejected at the .05 level of significance. This result indicates that it is necessary to take adequate account of factor prices in questions concerning the structures of factor markets. Changes in relative factor rewards will alter the demand for factors to a significant extent. Studies using Leontief technological matrices can be regarded as providing first approximation answers at best.

It may be argued that the test for a Leontief technology should be tested against the maintained hypothesis of symmetry. Conditional upon the covariance matrix used and upon symmetry the hypothesis that factor rewards do not influence the input output coefficients was tested and rejected at the .001 level for all but three industries. The hypothesis is acceptable at the .01 level for Forestry (2), at the .001 level for Trade (8), and at the .1 level for Finance (9). If symmetry is imposed before estimating the covariance matrix then the hypothesis is acceptable for Forestry alone and only at the .01 level.

In summary, all of our restriction sets have been rejected with relatively few exceptions. From a theoretical viewpoint the rejection of the symmetry restrictions is most significant.

4.3 Demand Elasticities for Labour

The last column of Table 2 contains the elasticity of demand for labour with respect to the wage rate, evaluated at 1961 factor rewards, output and technology index (time). The estimates derived from the unconstrained model and from the model under symmetry restrictions are provided.

TABLE 2

CROSS AND DIRECT ELASTICITIES
OF DEMAND FOR LABOUR

	BS	FM	PV	CV	L
1. Agriculture	.03 -.0007	-.27 .10*	.50 .007	.63 .04*	-.89 -.15
	ME	V	B	C	L
3. Fishing	-.43 -.30*	.28 .20*	.58 .018*	.20 .004	-.63 -.16
	BS	ES	ME	CC	L
2. Forestry	.13 -.0004	-.35 -.0001	.85* .019*		-.63 -.02
4. Mining	5.02* .096*	-4.01* .18*	-.82* .17*		-.19 -.45
5. Manufacturing	-.57* .04*	.79* .008	-1.12* .05*	1.40* .04*	-.49 -.13
6. Construction	.84* .0003	-1.02* -.0003	.29 .14*		-.11 -.14
7. Transportation	.63 -.01*	-.70 -.02	.36 .04*	-.71 .01*	.42 -.02
8. Trade	.17 .007	.03 .0006	-.25 .004	.08 .002	-.03 -.01
9. Finance	-.29* .18*		.14* .02*		.15 -.20
10. Services	-.40* .02*		.70* .02*		-.30 -.04

Note: Elasticities of the estimated demand functions are evaluated at 1961 factor rewards, output, and technology index (time). The estimates for the unconstrained model are followed underneath by those for the symmetry constrained model. The * denotes significance at the .1 level for the conditional estimates (not given for labour). The column label L denotes the factor labour. For a description of the other labels see section 3.3 of the text.

For two industries, Transportation and Finance, the labour demand elasticities are positive contrary to expectations. However once symmetry is imposed negative elasticities are obtained.

One of the striking features of the results is the inelasticity of the estimated labour demand functions. The most elastic case is Agriculture where the elasticity is just -0.89 and, in the restricted model the most elastic case is Mining where it is even lower at -.45. One explanation for these "low" results is that we cannot expect the demand for labour as a whole to be inelastic. This is because labour is really made up of various types and some types are complementary with capital, and, the more complementary labour is with capital the lower will be its own price elasticity.

The results indicate that perhaps labour as a whole can benefit by exercising monopoly powers in the labour market as long as output is not adversely affected and as long as induced technological change does not react against labour. In addition the results indicate that large changes in wage rates may be required to induce increases in labour demand required to reduce unemployment. In an economy such as Canada, where wages tend to be flexible only in an upward direction inelastic labour demand functions could therefore mean prolonged unemployment.

The elasticity of demand for labour with respect to changes in the rentals on the various capital types is also provided in Table 2 for both the unconstrained and the symmetry constrained models. Again we see that the elasticities are smaller in absolute value in the symmetry constrained model than in the unconstrained model, while they often change sign. This is due to the fact mentioned previously, that the unconstrained model often yielded estimates such that sign $\hat{\beta}_{ij} \neq$ sign $\hat{\beta}_{ji}$. Forcing the equality of these estimates

tends to push them towards zero. The symmetry constrained estimates show that the demand for labour is inelastic with respect to all factor rewards and generally most elastic with respect to the wage rate in all industries.

If we interpret the estimates as conditonal upon the factor rewards and the input-output coefficients[6] then the hypothesis that an elasticity is zero may be tested using the student "t" distribution. Table 2 indicates that labour demand is related significantly to the rentals of the various capital types in the following way. Labour is substitutable for machinery and equipment in all industries except Fishing and Trade using the symmetry constrained model, a result which one would expect at the industry level. However, in Mining and Manufacturing the unconstrained model shows significant complementarity. One explanation may be that skilled labour is complementary with machinery and equipment and that this effect dominates any substitution with unskilled labour. In Fishing, labour is a substitute for vessels (V) and boats (B) but is complementary to machinery and equipment. The demand for labour seems to be negatively related (complement) to rentals on building structures in Transportation, Finance, and Sales which is not unexpected. In Mining the relationship is positive while in Manufacturing the two estimates conflict as to sign. The influence of rentals on engineering structures upon the demand for labour is mixed in sign for Mining, positive for Manufacturing, and negative for Construction.

In summary, it appears that the influence of capital rentals upon the demand for labour differs from industry to industry in both the sign of the effect and in its significance. If this is true then it is important that demand functions for labour be estimated at the lowest possible level of aggregation.

4.4 Substitution Elasticities

Table 3 provides the estimates of the Hicks-Allen elasticities of substitution for 1961 together with the estimated factor shares. The significance of the elasticities, interpreted as conditional elasticities, is also indicated by the asterisk.

We wish to have the estimated cost function concave and monotone in factor rewards throughout its domain. None of the estimated cost functions exhibit these properties since not all of the off diagonal coefficients (hence substitution elasticities) are non-negative in any industry. However, monotonicity is obtained at all sample points as is indicated by the non-negative factor input predictions (not recorded here). Monotonicity was not obtained however, when symmetry was imposed before estimating the disturbance covariance matrix as mentioned above. Concavity at the 1961 sample points was obtained in three industries - Forestry, Finance, and Services. When symmetry was imposed from the beginning concavity was satisfied in 1961 for these three industries and for Mining.

We have already discussed labour-capital substitution so here we concentrate on capital-capital substitution. In Agriculture we get the result that vehicles are complements to farm machinery and very strong (σ_{ij} = 4.74) substitutes among themselves. In Fishing gear is a substitute for, while boats are a complement to, machinery and equipment. It appears that in most industries machinery and equipment substitutes for engineering structures where the latter occurs. The relationship between machinery and equipment and building structures is one of substitutability in Forestry and Trade and one of complementarity in Mining and in Finance.

One interesting feature of the substitution elasticity estimates is the prevalence of apparently (no tests have been undertaken) strong substitution and complementary forces. By strong it is meant that elasticities are

TABLE 3

HICKS-ALLEN ELASTICITIES OF SUBSTITUTION AND FACTOR SHARES - 1961 ESTIMATES

1. AGRICULTURE

BS	FM	PV	CV	L
-1.24	.31	.47	.10	-.01*
	-1.10	-3.68*	-2.8*	.64
		5.99	4.74*	.16*
			-5.22	.74
				-.21
.053	.164	.040	.047	.696

2. FORESTRY

BS	ES	ME	L
-32.85	4.72	3.80*	-.03
	-1.51	-.08	-.003
		-2.92	.22*
			-.02
.015	.039	.083	.863

(concave cost function)

3. FISHING

ME	V	B	G	L
13.49	-2.75	-5.36*	4.94*	-.87*
	-1.39	-3.88	-.98	1.75*
		6.88	-.72	-.15*
			.11	.02
				-.30
.034	.115	.123	.197	.531

4. MINING

BS	ES	ME	L
7.31	-1.82*	-2.92*	1.33*
	-1.48	2.26*	.55*
		-5.72	1.02*
			-1.05
.072	.330	.170	.428

5. MANUFACTURING

BS	ES	ME	OC	L
1.43	-11.16*	.83	-12.74*	.39*
	23.53	2.09	-8.79	.30*
		-1.41	-1.14	.24*
			14.68	2.18*
				-.20
.098	.028	.195	.019	.660

6. CONSTRUCTION

BS	ES	ME	L
-10.90	-23.94*	1.81*	.02
	-42.06	5.06	.14*
		-8.59	1.16
			-.16
.017	.002	.121	.860

7. TRANSPORTATION

BS	ES	ME	CC	L
-66.91	1.45	.73*	148.59*	-.96*
	-.94	2.45*	-9.58*	-.16*
		-4.6	-3.73	.32*
			105.27	1.54
				-.02
.011	.151	.125	.007	.707

8. TRADE

BS	ES	ME	OC	L
-2.04	.31	1.61*	16.72*	.05
	-20.96	2.90*	-16.11*	.05
		-1.83	-22.65	.04
			-59.74	.30
				-.02
.135	.012	.087	.005	.761

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TABLE 3 - CONTINUED

9. FINANCE	BS	ME	L	10. SERVICES	BS	ME	L
	-.21	-.14*	.32*		-1.35	-.33	.19*
		-.67	.39*			-9.22	.53*
			-.50				-.05
	.559	.047	.394		.109	.044	.847
	(concave cost function)				(concave cost function)		

Note: Only the upper triangle of the matrix of substitution elasticities is given because of its symmetry. The estimated factor shares in 1961 are given below the substitution matrix while the * denotes elasticities significantly different from zero at the .1 level and is provided only for off diagonal elements.

greater than unity or less than minus one. The implication of a $\sigma_{ij} > 1$ is that an increase in the rental on factor j will cause an increase in the share accruing to factor $i \neq j$ while factor i suffers a reduction in its share if $\sigma_{ij} < 1$. The general relationship is

.1 $$\frac{\partial\theta_i}{\partial w_j}\frac{w_j}{\theta_i} = (\sigma_{ij}-1)\,\theta_j + \delta_{ij} = \varepsilon_{ij} + (\delta_{ij}-\theta_j) \qquad i,j = 1,\ldots,n$$

where δ_{ij} is the Knonecker delta. In five industries there is at least one factor whose elasticity of substitution with labour is greater than unity. Increases in the rentals on these factors will therefore increase the share of the pie accruing to labour (and vice versa). As an example where labour might increase its share by demanding higher wages consider the Manufacturing industry where the direct elasticity of demand for labour is (symmetry assumed) -.13 from Table 2 while from Table 3 labour's estimated share is 0.66. Since $\varepsilon_{ii} + (1-\theta_i) = -.13 + (1-.66) > 0$ labour can increase its share through a higher wage rate. A similar result holds for all other industries with the exception of Construction where wage changes have no effect upon labour's share.

It is of some interest to compare the present results for Manufacturing with those obtained by Kotowitz (1968) using a CES (constant elasticity of substitution) production function estimated for Canada over the period 1946-1961. The conclusion of Kotowitz (1968, p. 623) is that "the value of the elasticity of substitution for total manufacturing is between .3 and .5 and is significantly different from both zero and one." There are at least two problems involved in a comparison of this result with our results. Here the elasticities of substitution are not independent of factor prices as with the CES model, and also the present study involves not one but several types of capital. We simply take the estimates for the 1961 sample points for comparison and define an aggregate

elasticity of substitution between capital and labour as

.2 $$\sigma_{KL} \equiv - \sigma_{LL}\, \theta_L \Big/ (1-\theta_L) = - \sum_{j \neq L} \sigma_{Lj}\, \theta_j \Big/ (1-\theta_L)$$

where the subscript L denotes labour, the subscript K denotes aggregate capital and θ_i is the share of factor i in the total cost.[7] This definition states that the aggregate elasticity is a weighted average of the individual capital-labour substitution elasticities and it satisfies the relation $\sigma_{LL}\, \theta_L + \sigma_{LK}\, \theta_K = 0$.

Calculation of σ_{LK} according to 4.2 yields $\sigma_{LK} = 0.39$ which is not inconsistent with the elasticity of substitution estimated by Kotowitz. But it camouflages the fact that there is a very strong substitution between labour and operating capital and a smaller substitution between labour and engineering structures.

Estimates of the aggregate capital-labour elasticity of substitution for the various industries in 1961 are provided in Table 4. All of the estimates are between zero and unity and seem to be highest for Construction and Mining and very low in Transportation and Trade.

5. Summary

Factor demand functions and hence the cost function and the production function were estimated using annual data for ten industrial divisions in Canada. The results indicate very low elasticities of demand for labour with respect to all factor rewards. Also the results show that labour as a whole can raise its share of the factor payments by raising the wage rate relative to the rentals on capital. In general technological change has played a significant role in altering the input-output coefficients while the scale of operations is indicated by the output level has also been significant. Finally, the existence of industry production functions seems doubtful in light of the rejection of the hypothesis of symmetry though this conclusion must be qualified because of the aggregation of labour to the one

TABLE 4

AGGREGATE CAPITAL-LABOUR

SUBSTITUTION ELASTICITIES

1.	Agriculture	.48	6.	Construction	.98
2.	Forestry	.13	7.	Transportation	.05
3.	Fishing	.34	8.	Trade	.06
4.	Mining	.79	9.	Finance	.33
5.	Manufacturing	.39	10.	Services	.28

category used in this study. This rejection together with other consequences of imposing symmetry suggest that economists should be wary in applying microeconomic theory to aggregate data.

APPENDIX A

The following tables give the parameter estimates together with their Student "t" values for the model estimated with the symmetry restrictions imposed. The last two rows in each coefficient matrix refer to the output and the time variables in each factor demand equation.

The factors are labelled as follows:

BS - building structures	PV - passenger vehicles
ES - engineering structures	CV - commercial vehicles
ME - machinery and equipment	FM - farm machinery
OC - operating capital	V - vessels
L - Labour	B - boats
	G - gear

1. Agriculture	BS .0653 (31.79) $-.2206 \div 10^4$ (22.49) .0011 (14.67)	FM .0058 (1.12) .2600 (8.21) $-.7100 \div 10^4$ (4.09) -.0025 (1.42)	PV .0021 (.51) -.0518 (3.57) .0641 (4.27) $-.8648 \div 10^5$ (1.69) 0.0008 (1.08)	CV .0005 (.15) -.0470 (3.34) .0193 (1.67) .0619 (2.95) $-.1558 \div 10^4$ (3.41) -.0008 (1.29)	L -.0010 (.65) .1563 (4.44) .0098 (.86) .0524 (5.03) 1.5024 (10.27) -.0005 (9.48) -.0110 (2.84)
2. Forestry	BS .0108 (1.54) $-.5334 \div 10^4$ (6.57 .0006 (6.82)	ES .0046 (1.20) .0393 (9.81) -.0001 (11.22) .0018 (19.34)	ME .0079 (2.45) -.0004 (.11) .0807 (4.85) -.0003 (4.37) .0033 (5.61)	L -.0006 (.30) -.0002 (.09) .0271 (2.05) 1.3117 (7.78) -.0006 (.94) -.0238 (3.53)	

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3. Fishing	ME .0612 (4.77) -.0002 (4.61) .0008 (6.77)	V -.0168 (1.10) .0053 (.08) .0012 (2.98) -.0032 (3.44)	B -.0352 (2.81) -.0854 (1.61) .2922 (4.73) -.0005 (1.98) .0007 (1.06)	G .0519 (10.99) -.0346 (1.19) -.0272 (1.08) .2300 (4.69) -.0006 (1.21) -.0012 (1.08)	L -.0245 (17.82) .1653 (13.11) -.0148 (1.74) .0034 (.21) .1861 (1.70) .0021 (1.49) -.0060 (2.60)
4. Mining	BS .1210 (4.26) .3735 ÷ 10^4 (2.22) -.0023 (1.64)	ES -.0803 (2.61) -.0819 (1.25) .0001 (2.37) -.0075 (1.47)	ME -.0666 (3.73) .2363 (4.33) -.1281 (2.45) .6398 ÷ 10^4 (1.38) -.0071 (1.85)	L .0764 (10.16) .1441 (4.41) .1390 (7.43) .3178 (5.44) .0002 (1.77) -.0365 (3.55)	

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5. Manufacturing	BS .1620 (8.64) $-.2897 \div 10^5$ (4.32) -.0008 (1.86)	ES -.0603 (3.55) .0732 (3.45) $-.2758 \div 10^5$ (7.28) .0010 (3.24)	ME .0308 (1.08) .0223 (.71) .0931 (1.21) $-.1637 \div 10^4$ (11.41) .0089 (9.36)	OC -.0465 (2.22) -.0092 (.40) -.0083 (.15) .0571 (1.38) $.4053 \div 10^5$ (4.41) -.0041 (6.15)	L .0489 (4.67) .0107 (.97) .0596 (2.09) .0536 (2.59) .8245 (27.47) $.2059 \div 10^4$ (4.48) -.0338 (15.48)
6. Construction	BS .0170 (4.39) $-.6491 \div 10^5$ (3.50) .0004 (1.70)	ES -.0017 (1.84) .0031 (3.95) $-.7831 \div 10^6$ (2.78) $.1669 \div 10^4$ (.47)	ME .0069 (1.06) .0027 (2.56) -.0755 (2.40) $.5121 \div 10^4$ (2.49) -.0096 (3.70)	L .0005 (.13) -.0005 (.84) .2296 (6.53) .9724 (14.17) -.0002 (3.43) .0070 (.94)	

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7. Transportation	BS -.0077 (1.06) .1415 ÷ 10^5 (3.09) -.0002 (1.65)	ES .0047 (.71) .1174 (5.98) -.1298 ÷ 10^4 (2.96) .0030 (2.92)	ME .0019 (.22) .0925 (6.20) .0189 (.86) -.2600 ÷ 10^4 (12.76) .0049 (10.00)	OC .0232 (3.63) -.0211 (2.77) -.0068 (.61) .0024 (.25) -.1999 ÷ 10^5 (4.44) .0002 (1.20)	L -.0145 (4.48) -.0333 (1.34) .0563 (3.78) .0160 (4.21) 1.1948 (26.50) -.2486 ÷ 10^4 (1.82) -.0250 (7.91)
8. Trade	BS .1106 (6.61) -.3505 ÷ 10^4 (12.21) .0067 (8.21)	ES .0010 (.23) .0112 (2.45) -.3814 ÷ 10^5 (13.51) .0007 (8.22)	ME .0386 (2.65) .0060 (2.04) .0638 (4.46) -.3205 ÷ 10^4 (13.14) .0087 (13.03)	OC .0250 (4.76) -.0021 (1.20) -.0218 (4.24) .0062 (1.99) -.4042 ÷ 10^5 (6.30) .0009 (4.71)	L .0113 (.58) .0009 (.78) .0055 (.34) .0025 (.77) 1.0981 (19.02) -.7404 ÷ 10^4 (3.80) -.0009 (.20)

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9. Finance	BS -.8124 (12.83) .0008 (12.25) -.0666 (10.13)	ME -.0066 (2.22) .0131 (2.18) $-.6399 \div 10^5$ (1.38) .0026 (6.01)	L .1282 (2.91) .0134 (3.14) -.0854 (.82) .0002 (6.10) -.0264 (6.21)		
10. Service	BS .0519 (12.50) $-.5033 \div 10^5$ (4.40) .0028 (14.72)	ME -.0030 (.54) .0126 (1.94) $.2663 \div 10^5$ (2.88) -.0011 (7.23)	L .0327 (3.82) .0368 (5.10) .7439 (24.28) $-.3985 \div 10^4$ (4.77) .0105 (5.25)		

APPENDIX B

The following tables provide the unconstrained (ie. symmetry not imposed) for the demand functions together with their Student "t" values. The notes in Appendix A concerning the labelling apply here also

BS - building structures	PV - passenger vehicles
ES - engineering structures	CV - commercial vehicles
ME - machinery and equipment	FM - farm machinery
OC - operating capital	V - vessesl
L - labour	B - boats
	G - gear

1. Agriculture	BS	FM	PV	CV	L
	.0648	.0431	-.0033	.0051	.0399
	(21.73)	(.98)	(.24)	(.42)	(.50)
	-.001	-.0401	-.1480	-.0989	-.3686
	(.01)	(.18)	(2.63)	(1.82)	(.81)
	.0406	.7722	.1474	.2114	.6938
	(3.49)	(4.49)	(3.26)	(5.00)	(1.50)
	-.0340	-.6632	.0297	-.0970	.8676
	(1.59)	(2.08)	(.36)	(1.23)	(1.20)
	-.0010	.0928	.0171	.0395	-.7941
	(.49)	(1.83)	(1.19)	(2.74)	(1.62)
	-.2101 $\div 10^4$	-.2189 $\div 10^4$	.1091 $\div 10^4$	.3901 $\div 10^6$	-.0002
	(19.14)	(.95)	(1.63)	(.07)	(2.67)
	.0012	.0014	-.0007	-.0003	.0390
	(13.68)	(.43)	(.56)	(.26)	(4.05)

2. Forestry	BS	ES	ME	L
	.0194	-.0009	.1486	.2341
	(1.19)	(.05)	(1.36)	(.19)
	-.0039	.0384	.0841	-.6322
	(.42)	(3.10)	(1.35)	(.87)
	.0116	.0076	.0475	1.5235
	(1.49)	(.82)	(1.01)	(2.29)
	-.0065	-.0027	-.0095	-.0686
	(1.63)	(.63)	(.34)	(.13)
	-.4735 $\div 10^4$	-.9378 $\div 10^4$	-.0002	.0007
	(4.95)	(9.23)	(3.57)	(.83)
	.0007	.0017	.0042	-.0259
	(5.20)	(11.40)	(4.42)	(2.48)

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3. Fishing	ME .1051 (5.02) -.1229 (4.89) .0470 (2.27) .0022 (.28) -.0040 (1.88) -.9211 ÷ 10^4 (1.50) .0002 (1.62)	V -.1085 (.69) 1.0872 (5.24) -1.3387 (8.13) .4708 (7.59) .0231 (1.33) -.0012 (2.45) .0042 (3.20)	B .1528 (1.88) -.1873 (1.79) .1903 (2.35) .0087 (.26) -.0242 (2.66) -.0009 (3.32) .0023 (3.02)	G -.0743 (.43) .2514 (1.15) -.2831 (1.62) .4335 (4.58) -.0613 (3.42) -.0012 (2.18) .0009 (.49)	L -.4607 (.73) .3036 (.34) .6256 (.90) .2168 (.80) -.1352 (1.08) -.0022 (1.22) .0116 (2.46)
4. Mining	BS .1130 (1.57) -.1159 (1.69) -.0007 (.02) .0589 (4.90) .1797 ÷ 10^4 (.97) -.0007 (.45)	ES .8062 (2.97) -.9470 (3.76) .1678 (1.35) .1489 (3.29) .0001 (1.47) -.0014 (.25)	ME -.1926 (1.02) .2382 (1.34) .0619 (.64) .0893 (2.89) .1296 ÷ 10^4 (.25) -.0029 (.71)	L 3.4160 (6.76) -2.7272 (5.89) -.5608 (2.31) .3365 (3.09) .8413 ÷ 10^4 (.58) -.0152 (1.25)	

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5. Manufacturing	BS	ES	ME	OC	L
	.078	-.0644	.1882	-.1069	-.7557
	(1.25)	(2.07)	(1.35)	(1.41)	(1.96)
	.0714	.0984	-.0155	-.0725	1.0754
	(.92)	(2.89)	(.09)	(.77)	(2.12)
	-.1792	-.0372	-.1015	.1359	-1.5220
	(1.92)	(.83)	(.48)	(1.28)	(3.04)
	.1716	.0401	.0569	.0340	1.8921
	(2.41)	(1.20)	(.38)	(.45)	(4.99)
	-.0009	-.0005	.0584	.0739	.3910
	(.03)	(.03)	(.87)	(2.08)	(2.08)
	$-.3149 \div 10^5$	$-.2980 \div 10^5$	$-.1864 \div 10^4$	$.4682 \div 10^5$	$.1822 \div 10^4$
	(2.96)	(5.51)	(7.30)	(3.22)	(2.11)
	-.0008	.0012	.0109	-.0054	-.0344
	(.91)	(2.47)	(5.10)	(4.68)	(5.43)

6. Construction	BS	ES	ME	L
	.0495	.0036	-.4797	1.3669
	(3.27)	(1.64)	(2.56)	(2.24)
	-.0350	-.0019	.4136	-1.6625
	(2.53)	(.90)	(2.47)	(3.09)
	.0023	.0012	.1231	.4800
	(.29)	(1.00)	(1.34)	(1.63)
	.0052	.0005	.1069	.9275
	(1.08)	(.70)	(1.77)	(4.52)
	$-.5184 \div 10^5$	$-.6776 \div 10^6$	$4318 \div 10^4$	$-.7815 \div 10^4$
	(2.68)	(2.37)	(1.74)	(.85)
	.0003	$.5533 \div 10^5$	-.0083	-.0086
	(1.12)	(.16)	(2.67)	(.76)

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	BS	ES	ME	OC	L
7. Transportation	-.0181	-.1156	-.0465	.0173	.8841
	(.89)	(.51)	(.42)	(.93)	(1.45)
	.0276	.3104	.0988	-.0283	-.9839
	(1.21)	(1.23)	(.77)	(1.41)	(1.56)
	-.0319	-.0843	.1349	.0231	.5123
	(1.04)	(.27)	(.86)	(.87)	(.64)
	.0474	.0629	-.1427	-.0178	-1.0042
	(1.76)	(.22)	(.99)	(.74)	(1.37)
	-.0139	.0164	.1265	.0177	1.8490
	(2.25)	(.24)	(4.09)	(2.96)	(8.06)
	$.1706 \div 10^5$	$.1035 \div 10^4$	$-.2623 \div 10^4$	$-.1872 \div 10^5$	$-.5596 \div 10^4$
	(2.23)	(1.17)	(6.53)	(2.45)	(1.86)
	-.0005	.0005	.0043	.0002	-.0191
	(1.65)	(.15)	(2.69)	(.81)	(1.89)
8. Trade	BS	ES	ME	OC	L
	.0401	.0012	-.0575	.0185	.2583
	(.39)	(.22)	(.71)	(1.16)	(.54)
	.1492	.0139	.1889	.0177	.0510
	(1.30)	(2.37)	(2.07)	(1.12)	(.12)
	.0188	.0050	-.0287	-.0400	-.3903
	(.25)	(1.38)	(.52)	(4.23)	(1.68)
	-.0522	-.0044	-.0275	.0103	.1217
	(1.27)	(2.02)	(.84)	(1.49)	(.58)
	.0326	.0013	.0266	.0046	1.0769
	(1.36)	(1.02)	(1.40)	(1.29)	(7.70)
	$-.3587 \div 10^4$	$-.3942 \div 10^5$	$-.2863 \div 10^4$	$-.3402 \div 10^5$	$-.7539 \div 10^4$
	(5.95)	(11.61)	(5.91)	(3.03)	(1.84)
	.0068	.0008	.0071	.0006	-.0010
	(3.38)	(7.00)	(4.46)	(1.74)	(.08)

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9. Finance	BS	ME	L		
	-.5543	-.0153	-.2196		
	(5.24)	(4.93)	(4.16)		
	.1510	.0268	.1098		
	(3.25)	(4.30)	(4.34)		
	.8893	.0224	.4401		
	(9.05)	(5.01)	(3.40)		
	.0001	$-.1414 \div 10^4$	.0002		
	(1.02)	(2.95)	(3.61)		
	-.0145	.0033	-.0263		
	(1.51)	(7.19)	(6.02)		
10. Services	BS	ME	L		
	-.0167	-.0461	-.6541		
	(.81)	(3.66)	(5.08)		
	.0843	.0711	1.1399		
	(3.24)	(4.37)	(6.37)		
	-.0060	.0047	.1588		
	(.44)	(.45)	(1.34)		
	$-.1883 \div 10^5$	$.5195 \div 10^5$	$-.4654 \div 10^4$		
	(1.33)	(4.74)	(5.52)		
	.0031	-.0008	.0246		
	(14.89)	(4.73)	(7.28)		

FOOTNOTES

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1 See Diewert (1971), pp.495-496.

2 For a lucid account of the Zellner procedure within the context of a system of equations involving linear parameter restrictions see Thiel (1971).

3 The National Accounts were provided by Statistics Canada. Further revisions are currently (September 1972) being made so the accounts, when published, will not be the same as those used in this study.

4 These estimates were kindly provided by W.E. Diewert and K. Scott of the Department of Manpower and Immigration.

5 This information was kindly provided by L. David of Statistics Canada.

6 Parks (1971), p. 135, makes use of the concept of a conditional estimate in order to test elasticities of substitution. The same reasoning may be applied to elasticities of demand.

7 This seems a natural definition to make. It is well known that cost minimisation implies that $\sum_{j=1}^{n} \sigma_{ij}\theta_j = 0$ for all $i=1,\cdots,n$ so we would like any definition of the aggregate capital-labour substitution elasticity σ_{LK} to satisfy the analagous relation $\sigma_{LL}\theta_L + \sigma_{LK}\theta_K = 0$. Taking factor i as labour we have that $0 = \sum_{j=1}^{n} \sigma_{Lj}\theta_j = \sigma_{LL}\theta_L + \sum_{j\neq L}\sigma_{Lj}\theta_j = \sigma_{LL}\theta_L + (1-\theta_L)$ $\sum_{j\neq L}\sigma_{Lj}\theta_j/(1-\theta_L)$ from which definition 4.2 naturally arises. Definition 4.2 of the text is, of course, consistent with the concept of an aggregate capital-labour substitution elasticity in a situation where all capital rentals vary in strict proportion, that is when capital is a Hicksian composite factor.

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